**Ph.D. COMMON ENTRANCE TEST**

**SUBJECT – Data Analytics. & Mathematical Science**

**Roll No:**

**PART B**

**Duration: 60 minutes Maximum Marks: 50**

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| **Instructions:**   1. **This entrance test question paper is not to be taken out of the examination hall** 2. **Part B Question paper consists of Section A and Section B** 3. **Section A consists of 30 MCQs carrying 1 Mark each. Put a tick (√) mark against the correct answer in the box given.** 4. **Section B consists of Descriptive questions carrying 5 marks each. Restrict your answer to 500 words. Additional plain sheets have been attached to the question paper to answer Section B** |

**SECTION – A**

**Answer the following by ticking (√) against the correct answer in the box given: 30 X 1 = 30**

1. Let X = (R, T ), where T is the smallest topology on R in which all the singleton

sets are closed. Then, which of the following statements are TRUE?

1. [0, 1) is compact in X
2. X is not first countable
3. Neither a) nor b)
4. Both a) and b)
5. Let {ϕ0, ϕ1, ϕ2, · · · } be an orthonormal set in L2[−1, 1] such that ϕn = CnPn, where

Cn is a constant and Pn is the Legendre polynomial of degree n, for each n ∈ N∪{0}.

Then, which of the following statements are TRUE?

1. ϕ6(1) = 1
2. ϕ7(−1) = 1
3. ϕ7(1) =
4. ϕ6(-1) =
5. Let y = (α, −1)T , α ∈ R be a feasible solution for the dual problem of the linear programming problem

Maximize: 5x1 + 12x2

subject to: x1 + 2x2 + x3 ≤ 10

2x1 − x2 + 3x3 = 8

x1, x2, x3 ≥ 0.

Which of the following statements is TRUE?

1. α < 3
2. 3 ≤ α < 5.5
3. 5.5 ≤ α < 7
4. α ≥ 7
5. The work done by the force where and are unit vectors in and directions, respectively, along the upper half of the circle

x2 + y2 = 1 from (1, 0) to (−1, 0) in the xy-plane is

2. The first derivative of a function f ∈ C∞(−3, 3) is approximated by an interpolating polynomial of degree 2, using the data (−1, f (−1)), (0, f (0)) and (2, f (2)). It is found that f ′(0) ≈ −(2/3)f (−1) + αf (0) + βf (2). Then, the value of 1/αβ is
3. 3
4. 6
5. 12
6. Let (Q, d) be the metric space with d(x, y) = |x−y|. Let E = {p ∈ Q : 2 < p2 < 3}. Then, the set E is
7. closed but not compact
8. not closed but compact
9. compact
10. Neither compact nor closed
11. Let V be an n-dimensional vector space and let T : V → V be a linear transformation such that Rank T ≤ Rank T3. Then which one of the following statements is necessarily true?
12. Null Space (T) = Range (T)
13. Null space (T) Range (T)={0}
14. Null space (T) Range (T)=W, a subspace
15. Null space (T) Range (T)
16. Let f : (0, ∞) → R be defined by f (x) = sin(x3)/x , then f is
17. bounded and uniformly continuous
18. bounded but not uniformly continuous
19. not bounded but uniformly continuous
20. Neither bounder nor uniformly continuous
21. Suppose p is a degree 3 polynomial such that p (0) = 1, p (1) = 2, and p (2) = 5. Which of the following numbers cannot equal p (3)?
22. 0
23. 2
24. 6
25. 10
26. The set of real numbers in the open interval (0, 1) which have more than one decimal expansion is
27. empty
28. non-empty but finite
29. countably infinite
30. uncountable

1. How many zeroes does the function f (x) = ex − 3x2 have in R?
2. 0
3. 1
4. 2
5. 3
6. Consider a cube C centered at the origin in R3. The number of invertible linear transformations of R3 which map C onto itself is
7. 72
8. 48
9. 24
10. 12
11. The number of rings of order 4, up to isomorphism, is:
12. 1
13. 2
14. 3
15. 4
16. The matrix is:
17. diagonalizable
18. nilpotent
19. idempotent
20. none of these
21. Let A ∈ Mn(C), then is diagonalizable if and only if:

1. A=0
2. A=I
3. n=2
4. None of these
5. The number of ring homomorphisms from Z[x, y] to F2[x]/(x3 + x2 + x + 1) equals

2. 1
3. Let S be a collection of subsets of {1, 2, . . . , 100} such that the intersection of any two sets in S is non-empty. What is the maximum possible cardinality |S| of S?

1. 100
2. For n ≥ 1, let Sn denote the group of all permutations on n symbols. Which of the following statements is true?

1. S3 has an element of order 4
2. S4 has an element of order 6
3. S4 has an element of order 5
4. S5 has an element of order 6
5. Jacobi iteration on the system of equations:

taking 1.8, 0.8 and -1.2 as initial values give the answer.

1. 7,-9,5
2. 7,9,5
3. 7,7,5
4. None of these
5. A basic solution in which one or more basic variables vanish is called
6. feasible solution
7. degenerate solution
8. optimal solution
9. Basic solution
10. The equations are called
11. Initial conditions
12. constraints
13. non-negativity constraints
14. Objective function
15. If and then Wronskian is given by


19. The integrating factor of the linear differential equation is


23. The solution of the differential equation is

26. The number of irreducible polynomials of the form x2 + ax + b, with a, b in the field F7 of 7 elements is:
27. 7
28. 35
29. 21
30. 49
31. What is the last digit of 972013?
32. 1
33. 3
34. 7
35. 9
36. Let f : R → R be a differentiable function such that , then
37. f is bounded
38. f is unbounded
39. f is increasing
40. f’ is unbounded
41. In how many ways can the group Z5 act on the set {1, 2, 3, 4, 5}?
42. 5
43. 24
44. 25
45. 120
46. Let V be the vector space over R consisting of polynomials p(t) over R of degree less than or equal to 4. Let D: V → V be the linear operator that takes any polynomial p(t) to its derivative p′(t). Then the characteristic polynomial f (x) of D is

49. The rank of the matrix: is
51. 2
52. 3
53. 4

**SECTION – B**

**Answer any four of the following: 5 X 4 = 20**

1. Let X be a continuous random variable with probability density function

and let , then Find .

1. The Ranks of 15 students in two subjects A and B are given below; the two numbers within brackets denoting the rank of the same student in A and B respectively. (1,10), (2,7), (3,2), (4,6), (5,4), (6,8), (7,3), (8,1), (9,11), (10,15), (11,9), (12,5), (13,14), (14,12), (15,13). Use spearman’s Rank formula to find rank correlation coefficient.
2. Let be the three-dimensional subspace of with orthonormal basis where , and let . Find the distance from to .
3. Find all the poles of the function and compute over , where is the curve defined as .
4. For what values of k does the linear system x − 3z = −3, 2x + ky − z = −2, x + 2y + kz = 1

in unknowns x, y, z have no solution?

1. Suppose that the joint probability function of two random variables X and Y is

, for x = 1, 2, 3 and 0 < y < 1.

Find the variance of X.

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